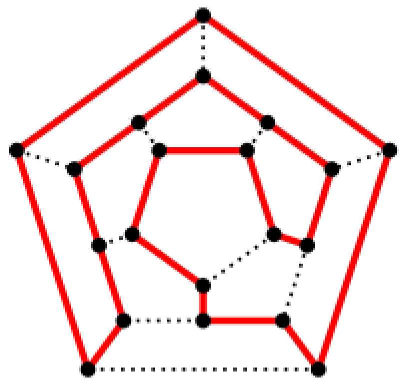
(1) Are the following problems in **PSPACE**? Why or why not?

- SAT
- SSP
- $PATH$
- $HAMPATH = \{\langle G, s, t\rangle \mid G$ is a directed graph with a Hamiltonian path from s to $t\}$
- $HAMCYCLE = \{\langle G\rangle \mid G$ is a directed graph with a Hamiltonian cycle$\}$

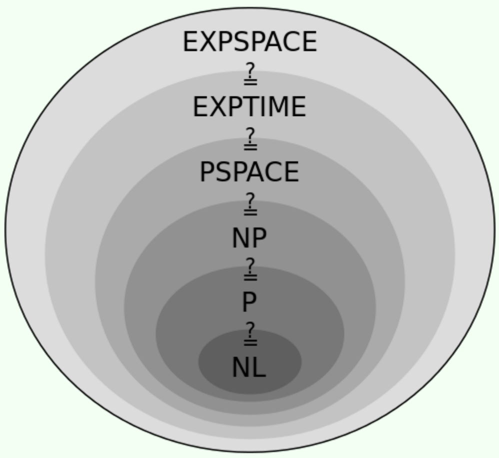
  Recall that "Hamiltonian cycle" means a "path that starts at a vertex, visits every vertex of G once, then returns back to the start vertex."

  Example:



Solution

All are in **PSPACE** because they are all in **NP**, and **NP** $\subset$ **PSPACE**.



(2) Show that the language of all binary strings with equal 0's and 1's is in **L**.

Solution

Read the input from left to right. Maintain a counter on the work tape with initial value zero, and increment or decrement it when reading "0" or "1" respectively.

Reject if the counter ever becomes negative, and accept if the counter is zero at the end of the input.

Since the counter can never exceed the input length n, it is an $O(log_2 n)$-bit number, so this only requires a work tape with $O(\log n)$ bits.

In general, any language which can be recognized by a machine with any constant number of counters is in **L**.

(3) Show that testing for balanced brackets is in **L**.

The corresponding language looks like:

$$\{\varepsilon, (), (()), ()(), ((())), ()()(), (())(), ()(()), \dots\}$$

Solution

This is the same as the previous one, just replacing 0's and 1's by ('s and)'s.

Read the input from left to right. Maintain a counter on the work tape with initial value zero, and increment or decrement it when reading "(" or ")" respectively.

Reject if the counter ever becomes negative, and accept if the counter is zero at the end of the input.

Since the counter can never exceed the input length n, it is an $O(log_2 n)$-bit number, so this only requires a work tape with $O(\log n)$ bits.

(4) Show that the language of palindromes over the alphabet $\{0, 1\}$ is in **L**.

Hint: simulate a **for**-loop that keeps track of the indices of the symbols that need to be compared.

Solution

To check that $w = w^R$, we check that each symbol matches the one in the position opposite to it from the end of the string, i.e. $w_i = w_{n-1-i}$ for $i = 0, 1, \dots, n-1$ where $n = |w|$.

A log-space TM can keep track of its current position by incrementing or decrementing a counter as it moves left or right. It can also carry out **for**/**while**-loops as long as the counters have values polynomial in n, since then the counters are $O(\log n)$-bit numbers and can fit in $O(\log n)$ space.