Basic

(1) Answer each part True or False, and briefly justify your answer.

- $2n = O(n)$
- $\log_{10} n = O(\log_2 n)$
- $n^2 = O(n)$
- $n^2 = O(n \log^2 n)$
- $n \log n = O(n^2)$
- $3^n = O(2^n)$
- $n! = O(n^n)$

(2) Given $f(n) = O(n^2)$ and $g(n) = O(n^3)$, what is the order of $f(n) + g(n)$, $f(n)g(n)$ and $f(g(n))$.

(3) Design an algorithm that, given a list of numbers, discovers if any number has occurred more than twice. (No need to write pseudocode – just the main idea.)

What is its cost? (Use O-notation).

Hint: There is an algorithm that costs $O(n^3)$ and a better one that only costs $O(n \log n)$.

(4) A **triangle** in an undirected graph is a 3-clique. Define the language

$$TRIANGLE = \{\langle G \rangle \mid G \text{ contains a triangle}\}$$

Show that $TRIANGLE \in \mathbf{P}$.

(5) A **Hamiltonian path** in a directed graph is a path that goes through each vertex exactly once.

$$HAMPATH = \{\langle G, s, t \rangle \mid \text{Directed graph } G \text{ has a Hamiltonian path from } s \text{ to } t\}.$$

Show that $HAMPATH \in \mathbf{NP}$.

(6) We say that two graphs G and H are **isomorphic** if the vertices of one of them can reordered to make identical to the other (i.e. their adjacency matrices become the same).

Define the language

$$ISO = \{\langle G, H \rangle \mid G \text{ and } H \text{ are isomorphic graphs}\}$$

Show that $ISO \in \mathbf{NP}$.