If there are any symbols or terminology you do not recognize then please let us know.

(1) Give the truth table for the following propositions

Expression	**Meaning**
$a \wedge b$	a and b
$a \vee b$	a or b
$a \oplus b$	a xor b
$\neg a$ (or $\bar{a}$)	not a
$a \implies b$	a implies b, or: if a then b
$a \iff b$	a and b are equivalent, or: "a if and only if b"

It is usual to apply these "bit-wise" to the bits of integers, e.g. `0011` $\oplus$ `0101` $=$ `0110`.

(2) Recall that:

- $\mathbb{N} = \{1, 2, 3, \ldots\}$ is the set of **natural numbers**
- $\mathbb{Z} = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$ is the set of **integers**.

Consider the following set definitions

- $A = \{a \in \{1, 2, 3, 4\} \mid (a < 2) \vee (a > 3)\}$
- $B = \{a \in \mathbb{N} \mid a < 9\}$
- $C = \{a \in \mathbb{N} \mid a > 2 \wedge a < 7\}$
- $D = \{i \in \mathbb{Z} \mid i^2 \leq 9\}$

a) Give an explicit enumeration for each set, i.e. write down the elements in the form $\{x_1, x_2, \ldots\}$.

b) What is the cardinality of each set?

c) Which of these sets are subsets of at least one other set?

(3) Write formal descriptions of the following sets.

a) The set containing all natural numbers that are less than 5.

b) The set containing all integers that are greater than 5.

c) The set containing the strings `aa` and `ba`.

d) The set containing the empty string.

e) The set containing nothing at all.

f) The set containing all the even integers.

(4) If the set A is $\{1, 3, 4\}$ and the set B is $\{3, 5\}$, write down:

Expression	**Meaning**
$A \cup B$	union of A and B
$A \cap B$	intersection of A and B
$A - B$	A minus B
$A \times B$	Cartesian product of A and B: set of all possible pairs (a, b) where $a \in A$ and $b \in B$
2^B (or $\mathcal{P}(B)$)	power set of B: set of all subsets of B

(5) Let X be the set $\{1, 2, 3, 4, 5\}$ and Y be the set $\{6, 7, 8, 9, 10\}$.

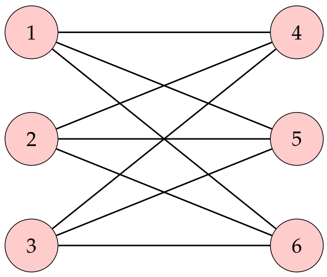
The *unary* function $f: X \to Y$ and the *binary* function $g: (X \times Y) \to Y$ are described in the following tables:

n	$f(n)$
1	6
2	7
3	6
4	7
5	6

g	6	7	8	9	10
1	10	10	10	10	10
2	7	8	9	10	6
3	7	7	8	8	9
4	9	8	7	6	10
5	6	6	6	6	6

- What are the *range* and *domain* of f?
- What are the *range* and *domain* of g?
- What is the value of $f(2)$?
- What is the value of $g(2, 10)$?
- What is the value of $g(4, f(4))$

(6) Write a formal description of the following graph.



(7) Draw the (undirected) graph $G = (V, E)$, where

$$V = \{1, 2, 3, 4, 5\}$$
$$E = \{(1, 2), (1, 4), (2, 3), (2, 4), (3, 5), (1, 5)\}$$

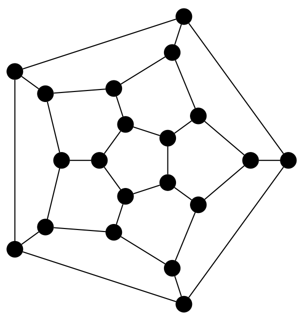
a) Is the graph connected?

b) What about the graph $G' = (V', E')$, where $V' = \{1, 2, 3, 4\}$ and $E' = \{(1, 3), (2, 4)\}$?

(8) Draw the graph $G = (V, E)$, where $V = \{1, \ldots, 5\}$ and

$$E = \{(a, b) \mid a, b \in V \wedge (a < b < a + 3)\}.$$

(9) The "Icosian Game" is a 19$^{\text{th}}$-century puzzle invented by the Irish mathematician Sir William Hamilton (1805–1865). The game was played on a wooden board with holes representing major world cities and grooves representing connections between them (see figure below).



The object is to find a cycle that would pass through all the cities exactly once before returning to the starting point. Can you find such routes?

Think about the following problems using pen-and-paper or "quick code experiments." Try to develop your solutions from theoretical analysis and experimental observations.

(1) Add the missing arithmetic operators ($+, -, \times, /$) and parentheses to the following expression to make it true:

$$3 \quad 1 \quad 3 \quad 6 \quad = \quad 8.$$

(2) A little girl counts from 1 to 1000 using the fingers of her left hand as follows. She starts by calling her thumb 1, the first finger 2, middle finger 3, ring finger 4, and little finger 5. Then she reverses direction, calling the ring finger 6, middle finger 7, the first finger 8, and her thumb 9, after which she calls her first finger 10, and so on. If she continues to count in this manner, on which finger will she stop?

(3) There are 100 closed lockers in a hallway. A man begins by opening all one hundred lockers. Next, he closes every second locker. Then he goes to every third locker and closes it if it is open or opens it if it is closed.

He continues like this until his 100^{th} pass in the hallway, in which he only changes the state of locker number 100.

How many lockers will be left open at the end?

(4) There are eight identical-looking coins; one of these coins is counterfeit and is known to be lighter than the genuine coins. What is the minimum number of weighings needed to identify the fake coin with a two-pan balance scale without weights?

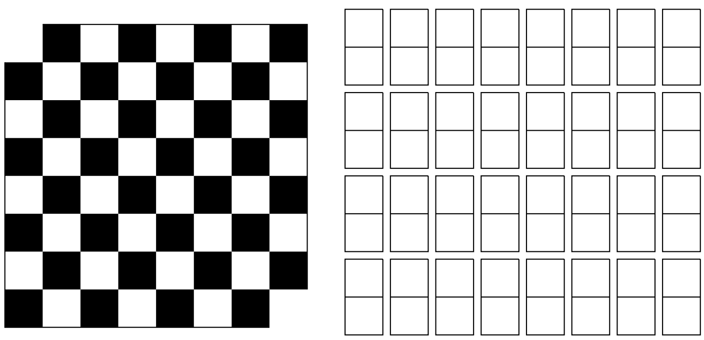
(5) You have a 5 litre jug and a 3 litre jug, and an unlimited supply of water, but no measuring cups.

How would you come up with exactly 4 litre of water?

(6) Little Alice has 10 pockets and £44 in £1 coins.

She wants to put her coins in her pockets so distributed that each pocket contains a different number of pounds. Can she do so?

(7) Below is an 8×8 chess board in which two diagonally opposite corners have been cut off.



You are given plenty of dominoes, such that each domino can cover exactly two squares.

Can you cover the entire board with dominoes? (No dominoes are allowed to overlap or be partly outside the board.)

Can you *prove* your answer is correct? (Show an example solution if this is possible, or show that it is impossible.)

PS. If you enjoy this kind of problems then have a look at Algorithmic Puzzles and Cracking the coding interview.